Problem set 4. Probabilistic models

Exercise 1. Sales of cars and light trucks. Motor vehicles sold to individuals are classified as either cars or light trucks (including SUVs) and as either domestic or imported. In a recent year, 69% of vehicles sold were light trucks, 78% were domestic, and 55% were domestic light trucks. Let A be the event that a vehicle is a car and B the event that it is imported.
Write each of the following events in set notation and give its probability.

(a) The vehicle is a light truck.
(b) The vehicle is an imported car.

	cars	light trucks	Total
domestic		55%	78%
imported			
Total		69%	

Exercise 2. Two dice are rolled. Describe the sample space S. Let A be an event of all outcomes $\omega =$"the sum of spots on two diece is odd" and let B be an event of all outcomes $\omega =$"'1' occurred at least on one die". Describe in set notation the following events: B, $A \cup B$, $A \backslash B$, $A \cap B$, $B \backslash A$,
for example $A = \{(k_1, k_2) : k_1 + k_2 \text{ is odd }, k_1, k_2 = 1, 2, \dots, 6\}$.

Exercise 3. A coin is tossed repeatedly until it falls twice in a row on the same side. What is a sample space S. Describe the following events

(a) the game ends before five toss,

(b) the game ends after the even number of tosses

(c) the coin never falled two times in a row on the same side.

Exercise 4. A coin is tossed until it falls twice in a row on the same side. Let $n \geq 2$ be a number of tosses it takes until this happens. Describe the outcomes ending in n-th toss and assign the them probabilities. Find the probability of the following events:

(a) the game ends not later than the sixth toss,

(b) the game ends after even number of tosses,

Exercise 5. Distribution of blood types. All human blood can be "ABO-typed" as one of O, A, B, or AB, but the distribution of the types varies a bit among groups of people. Here is the distribution of blood types for a randomly chosen person in the United States:

Blood type	A	B	AB	O
U.S. probability	0.40	0.11	0.04	?

(a) What is the probability of type O blood in the United States?
(b) Maria has type B blood. She can safely receive blood transfusions from people with blood types O and B. What is the probability that a randomly chosen American can donate blood to Maria?

Exercise 6. Blood types in China. The distribution of blood types in China differs from the U.S. distribution given in the previous exercise:

Blood type	A	B	AB	O
China probability	0.27	0.26	0.12	0.35

Choose an American and a Chinese at random, independently of each other. What is the probability that both have type O blood? What is the probability that both have the same blood type?

Exercise 7. Rh blood types. Human blood is typed as O, A, B, or AB and also as Rh-positive or Rh-negative. ABO type and Rh-factor type are independent because they are governed by different genes. In the American population, 84% of people are Rh-positive. Use the information about ABO type in Exercise 5 to give the probability distribution of blood type (ABO and Rh) for a randomly chosen person.

Exercise 8. Are the probabilities legitimate? In each of the following situations, state whether or not the given assignment of probabilities to individual outcomes is legitimate, that is, satisfies the rules of probability. If not, give specific reasons for your answer.
(a) Choose a college student at random and record gender and enrollment status:

P(femalefull-time) = 0.46, P(female part-time) = 0.54,
P(male full-time) = 0.44, P(male part-time) = 0.56.

(b) Deal a card from a shuffled deck:

$P(\text{clubs}) = 12/52, \quad P(\text{diamonds}) = 12/52,$
$P(\text{hearts}) = 12/52, \quad P(\text{spades}) = 16/52.$

(c) Roll a die and record the count of spots on the up-face:

$P(1) = 1/3, \quad P(2) = 1/6, \quad P(3) = 0,$
$P(4) = 1/3, \quad P(5) = 1/6, \quad P(6) = 0.$

Exercise 9. French and English in Canada. Canada has two official languages, English and French. Choose a Canadian at random and ask, "What is your mother tongue?" Here is the distribution of responses, combining many separate languages from the broad Asian/Pacific region:

Language	English	French	Asian/Pacific	Other
Probability	?	0.23	0.07	0.11

(a) What probability should replace "?" in the distribution?
(b) What is the probability that a Canadian's mother tongue is not English?

Exercise 10. Spam topics. A majority of email messages are now "spam." Choose a spam email message at random. Here is the distribution of topics:

Topic	Adult	Financial	Health	Leisure	Products	Scams
Probability	0.145	0.162	0.073	0.078	0.210	0.142

(a) What is the probability that a spam email does not concern one of these topics?
(b) Corinne is particularly annoyed by spam offering "adult" content (that is, pornography) and scams. What is the probability that a randomly chosen spam email falls into one or the other of these categories?

Exercise 11. Race in the census. The 2000 census allowed each person to choose from a long list of races. That is, in the eyes of the Census Bureau, you belong to whatever race you say you belong to. "Hispanic/Latino" is a separate category; Hispanics may be of any race. If we choose a resident of the United States at random, the 2000 census gives these probabilities:

	Hispanic	Not Hispanic
Asian	0.000	0.036
Black	0.003	0.121
White	0.060	0.691
Other	0.062	0.027

Let A be the event that a randomly chosen American is Hispanic, and let B be the event that the person chosen is white.
(a) Verify that the table gives a legitimate assignment of probabilities.
(b) What is $P(A)$?
(c) Describe B^c in words and find $\mathrm{P}(B^c)$ by the complement rule.
(d) Express "the person chosen is a non-Hispanic white" in terms of events A and B. What is the probability of this event?

Exercise 12. Are the events independent? The previous exercise assigns probabilities for the ethnic background of a randomly chosen resident of the United States. Let A be the event that the person chosen is Hispanic, and let B be the event that he or she is white. Are events A and B independent? How do you know?

Exercise 13. Winning the lottery. A state lottery's Pick 3 game asks players to choose a three-digit number, 000 to 999. The state chooses the winning three-digit number at random, so that each number has probability $1/1000$. You win if the winning number contains the digits in your number, in any order.
(a) Your number is 456. What is your probability of winning?
(b) Your number is 212. What is your probability of winning?

Exercise 14. Show that

1. $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$.
2. $\mathrm{P}(A \cup B \cup C) = \mathrm{P}(A) + \mathrm{P}(B) + \mathrm{P}(C) - \mathrm{P}(A \cap B) - \mathrm{P}(A \cap C) - \mathrm{P}(B \cap C) + \mathrm{P}(A \cap B \cap C)$.
3. Generalize the formula above to $\mathrm{P}(\bigcup_{i=1}^{n} A_i)$.