

Problem set 06. Conditional Probability.

Exercise 1. Spelling errors. Spelling errors in a text can be either nonword errors or word errors. Nonword errors make up 25% of all errors. A human proofreader will catch 90% of nonword errors and 70% of word errors. What percent of all errors will the proofreader catch? (Draw a tree diagram to organize the information given.)

Exercise 2. We toss a balanced coin four times.

1. Determine the probabilities of getting 0, 1, 2, 3, and 4 heads.
2. The following events are given:
 - A - four heads,
 - B - an even number of heads,
 - C - more heads than tails.

Determine the probabilities of the following events:

$$P(A), P(B), P(C), P(A|B), P(B|A), P(A|C), \\ P(C|A), P(B|C), P(C|B).$$

Exercise 3. In a factory, three workers deliver the same parts, with 30% of the parts coming from worker I, 45% from worker II, and the rest from worker III. The workers produce 2%, 1%, and 3% defects, respectively. What is the probability that a randomly selected part will turn out to be good?

Exercise 4. Consider families with three children and assume that each child is equally likely to be a boy or a girl. If such a family is picked at random and the eldest child is found to be a boy, what is the probability that the other two are girls? The same question if a randomly chosen child from the family turns out to be a boy.

Exercise 5. All the screws in a machine come from the same factory but it is as likely to be from Factory A as from Factory B. The percentage of defective screws is 5% from A and 1% from B. Two screws are inspected; if the first is found to be good, what is the probability that the second is also good?

Exercise 6. Greg and Johnny alternately flip a coin. Johnny wins when the (H, H, T) appears, and Greg wins when the (T, H, T) appears. What are the probabilities of winning for each boy?

Exercise 7. A white ball is dropped into an urn containing a total of n balls, each containing white and black balls. What is the probability of drawing a white ball from the urn if all hypotheses regarding the original composition of the urn are equally probable?

Exercise 8. Based on life tables, we know that 89.835% of women live to age 60, and 57.062% to age 80. What is the probability that Mrs. Ala, who is 60, will live another twenty years?

Exercise 9. From a batch of five items, one item was randomly selected and turned out to be defective. Any number of defective items of 0, 1, 2, 3, 4 or 5 in a batch is equally probable. What is the probability that there were more than 3 defective items in the batch?

Exercise 10. Among the 100 coins there is one coin with two heads. One coin was drawn and then tossed 6 times, getting heads each time. What is the probability that the chosen coin had two heads?