Problem set 07. Probability distribution of the discrete random variable.

Exercise 1. The distribution function of the random variable X has the form

x	$(-\infty,-1]$	$(-1,3]$	$(3,7]$	$(7,10]$	$(10,15]$	$(15,30]$
$F(x)$	0	0.15	0.25	0.4	0.85	1

a) Determine the probability distribution of variable X
b) Calculate $\mathrm{P}(0 < X < 7)$, $\mathrm{E}[X]$, and $\mathrm{D}^2[X]$.

Exercise 2. The random variable X has the probability distribution given in the table

x_i	-3	-2	-1	0	1	2
p_i	0.1	0.2	0.2	0.3	p	0.1

Give the random probability distribution of the variable $Y = X^2 + 2$.

Exercise 3. The random variable X for which $\mathrm{P}(X = x_k) = p_k$ has the distribution given in the table:

x_k	1	2	4	4.5
p_k	0.2	0.4	0.3	α

Calculate α and plot the cumulative distribution function of random variable X and $\mathrm{P}(\sqrt{X} \leq 2)$ and $\mathrm{P}(\log_2 X < 3)$.

Exercise 4. The random variable X for which $\mathrm{P}(X = x_k) = p_k$ has the distribution given in the table: has the probability distribution given in the table

x_k	1	2	4
p_k	0.25	0.35	0.4

Calculate the median.

Note: The median me is any number satisfying the following conditions:
$\mathrm{P}(X \leq me) \geq 0.5$ and $\mathrm{P}(X \geq me) \geq 0.5$.

Exercise 5. Let $S = \{0, 1, 2, 3\}$ and $\mathrm{P}(\{\omega\}) = 1/4$ for each $\omega \in S$. Find the distribution and cumulative distribution function of the random variable $Y(\omega) = \cos(0.5\pi\omega)$.

Exercise 6. Let $p_n = \mathrm{P}(X = 2^n) = \alpha 5^{n-1}$, $n = 1, 2, 3, 4, \ldots$. Determine α. Calculate $\mathrm{E}[X]$ and $\mathrm{D}^2[X]$?

Exercise 7. Let $p_n = \mathrm{P}(X = 2^n) = \alpha\beta^{n-1}$, $n = 1, 2, 3, 4, \ldots$. For which values of α and β does this formula give the probability distribution of the random variable X? For which values of β do moments of order k exist?

Exercise 8. The game involves throwing pucks onto a peg. A player receives six pucks and throws them until the first successful throw is made. Find the probability that after throwing a puck onto the peg, the player will have at least one puck left if the probability of hitting the peg on each throw is 0.1.

Exercise 9. Items produced independently of each other come off the assembly line. A defective item can be produced with probability p, and a nondefective item with probability $q = 1 - p$. If the k-th defective item appears on the assembly line, we immediately stop the assembly line. Find the distribution of the number N of items produced until the assembly line is stopped.

Exercise 10. Let X denote the waiting time for a six to be rolled when rolling a die. Find the probability distribution of random variable X. Calculate the probabilities of rolling a six for the first time:

a) exactly on the tenth roll,

b) no later than the tenth roll,

c) no earlier than the eleventh roll.

Exercise 11. The player draws two cards from the deck (without replacement). If they are 2 aces, he/she wins 20\$, if they are two of the other face cards (king, queen, jack), he/she wins 10\$. In all other cases, the player pays 2\$. Let X denote the player's winnings. Find the distribution and cumulative distribution function of X.

Exercise 12. From a batch of 100 items, including 10 defective items, 5 are selected randomly without replacement. Let X denote the number of defective items in the sample. Find the distribution and cumulative distribution function of variable X.

Exercise 13. We roll a die twice. Let X_i be the number of dots in the i-th roll, $i = 1.2$. Find the distribution of the random variable $Z = |X_1 - X_2|$.

Note: The median me is any number satisfying the following conditions: $\mathrm{P}(X \leq me) \geq 0.5$ and $\mathrm{P}(X \geq me) \geq 0.5$.

Exercise 14. Calculate the median of a random variable X with a geometric distribution, i.e., such that $\mathrm{P}(X = k) = q^{k-1}p$, where $p + q = 1$, $p > 0$, $q > 0$, $k = 1, 2, 3, \ldots$.