Problem set 08. Probabilistic distribution of a continuous random variable

Exercise 1. For what values of parameters a, b, c, is the function

$$f(x) = \begin{cases} a & \text{for } x < -1 \\ b & \text{for } -1 \leq x < 1 \\ c & \text{for } x \geq 1 \end{cases}$$

the density of some random variable X?

Exercise 2. The density of random variable X is given by

$$f(x) = \begin{cases} \alpha\sqrt{x} & \text{for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Compute α, then $\mathrm{E}[X]$ and $\mathrm{D}^2[X]$.

Exercise 3. Random variable X has a distribution with density $f(x) = \alpha x$ for $x \in [0, \pi]$ and $f(x) = 0$ otherwise. Find α. For random variable $Y = \cos X$, calculate $\mathrm{E}[Y]$ without finding the distribution of variable Y.

Exercise 4. A statistics teaching assistant typically arrives at the classroom two minutes before the scheduled start time. Assuming the arrival time is a normally distributed random variable with $\sigma = 2$ minutes, what is the probability that this assistant will be late for class?

Exercise 5. Let random variables X and Y be independent. Let X have an exponential distribution with parameter $\lambda = 1/5$ and Y have a normal distribution $\mathcal{N}(-1, 2)$. Find the variance of the random variable $Z = 2X - 3Y - 2$.

Exercise 6. Random variables X_i, $i = 1, 2, 3, 4$ are independent and have the same distribution $\mathcal{N}(1, 1)$. Compute

$$\mathrm{P}(|X_1 + X_2 + X_3 + X_4| > 6).$$

Exercise 7. Let X be uniformly distributed on the interval $[-1, 2]$. Find the distribution function and the density of the random variable $Y = X^2$.

Exercise 8. Random variable X has an exponential distribution with expected value 1. Find the density of random variable $Y = \ln X$.

Exercise 9. A string of Christmas lights consists of n bulbs connected in series. It is known that the bulbs fail independently and the operating time of each bulb has an exponential distribution with parameter λ. Find the distribution of operating time T of the string of lights.

Exercise 10. A room is lit by two independently operating light bulbs. The light durations of bulbs X and Y have an exponential distribution with parameter λ. Let T denote the time when the last working bulb fails. Determine

a) the cumulative distribution function of variable T,

b) the density function of variable T,

c) $\mathrm{E}[T]$,

d) $\mathrm{D}^2[T]$.