Problem set 5. Probabilistic models

Exercise 1. A batch of 100 of manufactured items is subjected to random inspection. The entire batch is rejected if at least one defective item is found among the five inspected items. What is the probability of rejecting a given batch if it contains 5% of defective items?

Exercise 2. In the exam there are five questions accompanied with three possible answers but only one answer is correct. The student choose the answers to the questions at random. What is the probability that four answers will be correct?

Exercise 3. Three numbers x, y, z were chosen independently from the interval $[0, 1]$ regarding to the uniform distribution. What is the probability that their sum is greater then 1?

Exercise 4. Suppose that the probability that both twins are boys is α, and that both are girls β; suppose also that when the twins are of different sexes the probability of the first born being a girl is $1/2$. If the first born of twins is a girl, what is the probability that the second is also a girl?

Exercise 5. Three marksmen hit the target with probabilities $1/2$, $2/3$, $3/4$, respectively. They shoot simultaneously and there are two hits. Who missed? Find the probabilities.

Exercise 6. On a flight from Urbana to Paris my luggage did not arrive with me. It had been transferred three times and the probabilities that the transfer was not done in time were estimated to be $4/10$, $2/10$, $1/10$, respectively, in the order of transfer. What is the probability that the first airline goofed?

Exercise 7. Two dice are rolled, one die is red and the other is white. What is the probability of getting six spots on the red die and one spot on the white die if we are given that the sum of spots on the both dice is even.

Exercise 8. Two numbers x and y are chosen randomly and independently from the interval $[-1, 1]$. What is the probability that $x^2 + y^2 < 1$ if we are given that $x^2 + y^2 > 0.25$.

Exercise 9. Albinism. People with albinism have little pigment in their skin, hair, and eyes. The gene that governs albinism has two forms (called alleles), which we denote by a and A. Each person has a pair of these genes, one inherited from each parent. A child inherits one of each parent's two

alleles, independently with probability 0.5. Albinism is a recessive trait, so a person is albino only if the inherited pair is aa.

(a) Beth's parents are not albino but she has an albino brother. This implies that both of Beth's parents have type Aa. Why?

(b) Which of the types aa, Aa, AA could a child of Beth's parents have? What is the probability of each type?

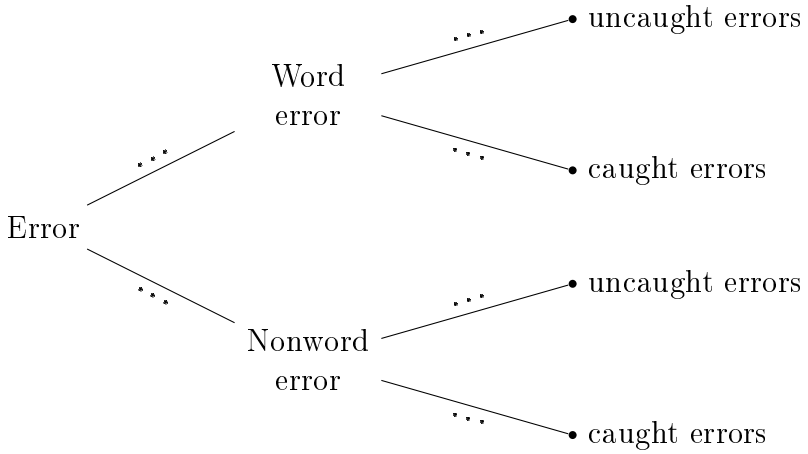
(c) Beth is not albino. What are the conditional probabilities for Beth's possible genetic types, given this fact? (Use the definition of conditional probability.)

Exercise 10. Find some conditional probabilities. Beth knows the probabilities for her genetic types from part (c) of the previous exercise. She marries Bob, who is albino. Bob's genetic type must be aa.

(a) What is the conditional probability that a child of Beth and Bob is non-albino if Beth has type Aa? What is the conditional probability of a non-albino child if Beth has type AA?

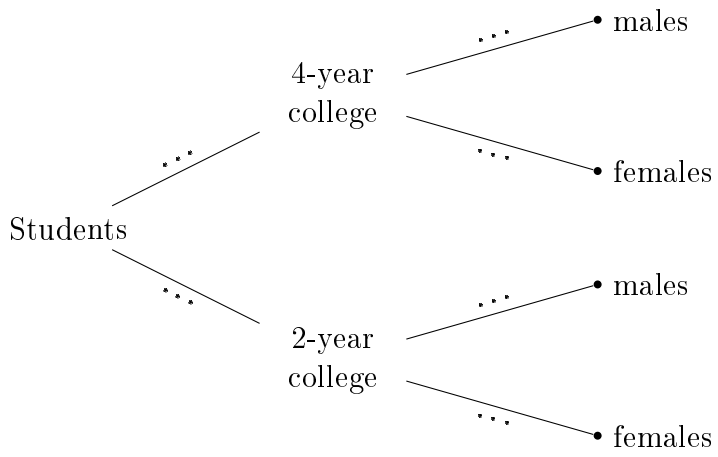
(b) Beth and Bob's first child is non-albino. What is the conditional probability that Beth is a carrier, type Aa?

Exercise 11. Spelling errors. Spelling errors in a text can be either nonword errors or word errors. Nonword errors make up 25% of all errors. A human proofreader will catch 90% of nonword errors and 70% of word errors. What percent of all errors will the proofreader catch? (Draw a tree diagram to organize the information given, for example, fill the diagram below.)

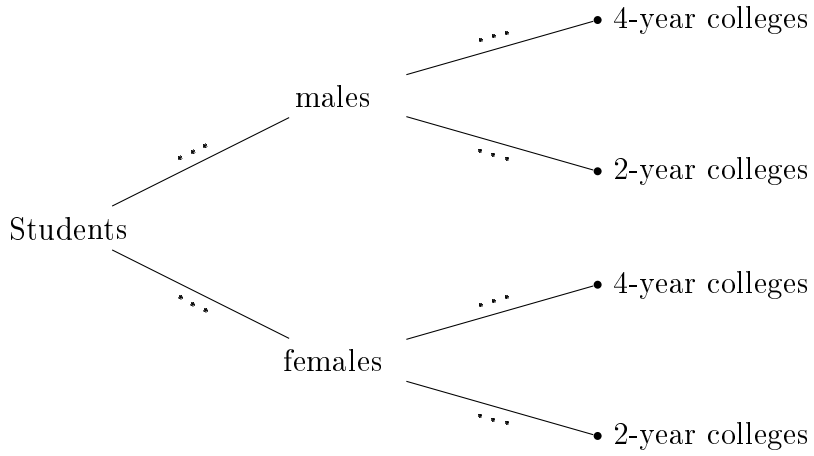


Exercise 12. Attendance at 2-year and 4-year colleges. In a large national population of college students, 61% attend 4-year institutions and the rest

attend 2-year institutions. Males make up 44% of the students in the 4-year institutions and 41% of the students in the 2-year institutions.
Draw a tree diagram. Draw a tree diagram to illustrate the probabilities in a situation where you first identify the type of institution attended and then identify the gender of the student.



Exercise 13. Draw a different tree diagram for the same setting. Refer to the previous exercise. Draw a tree diagram to illustrate the probabilities in a situation where you first identify the gender of the student and then identify the type of institution attended. Explain why the probabilities in this tree diagram are different from those that you used in the previous exercise.



Exercise 14. Wine tasters. Two wine tasters rate each wine they taste on a scale of 1 to 5. From data on their ratings of a large number of wines, we obtain the following probabilities for both tasters' ratings of a randomly

chosen wine:

	Taster 2					
Taster 1	1	2	3	4	5	
1	0.03	0.02	0.01	0.00	0.00	
2	0.02	0.07	0.06	0.02	0.01	
3	0.01	0.05	0.25	0.05	0.01	
4	0.00	0.02	0.05	0.20	0.02	
5	0.00	0.01	0.01	0.02	0.06	

(a) Why is this a legitimate assignment of probabilities to outcomes?

(b) What is the probability that the tasters agree when rating a wine?

(c) What is the probability that Taster 1 rates a wine higher than 3? What is the probability that Taster 2 rates a wine higher than 3?

Exercise 15. Cystic fibrosis. Cystic fibrosis is a lung disorder that often results in death. It is inherited but can be inherited only if both parents are carriers of an abnormal gene. In 1989, the CF gene that is abnormal in carriers of cystic fibrosis was identified. The probability that a randomly chosen person of European ancestry carries an abnormal CF gene is 1/25. (The probability is less in other ethnic groups.) The CF20m test detects most but not all harmful mutations of the CF gene. The test is positive for 90% of people who are carriers. It is (ignoring human error) never positive for people who are not carriers. Jason tests positive. What is the probability that he is a carrier?

Exercise 16. Use Bayes's rule. Refer to the previous exercise. Jason knows that he is a carrier of cystic fibrosis. His wife, Julianne, has a brother with cystic fibrosis, which means the probability is 2/3 that she is a carrier. If Julianne is a carrier, each child she has with Jason has probability 1/4 of having cystic fibrosis. If she is not a carrier, her children cannot have the disease. Jason and Julianne have one child, who does not have cystic fibrosis. This information reduces the probability that Julianne is a carrier. Use Bayes's rule to find the conditional probability that Julianne is a carrier, given that she and Jason have one child who does not have cystic fibrosis.

Working. In the language of government statistics, you are "in the labor force" if you are available for work and either working or actively seeking work. The unemployment rate is the proportion of the labor force (not of the entire population) who are unemployed. Here are data from the Current

Population Survey for the civilian population aged 25 years and over. The table entries are counts in thousands of people.

Highest education	Total population	In labor force	Employed
Did not finish high school	28,021	12,623	11,552
High school but no college	59,844	38,210	36,249
Some college, but no bachelor's degree	46,777	33,928	32,429
College graduate	51,568	40,414	39,250

Exercise 17. Conditional probabilities and independence.
(a) What is the probability that a randomly chosen person 25 years of age or older is in the labor force?
(b) If you know that the person chosen is a college graduate, what is the conditional probability that he or she is in the labor force?
(c) Are the events "in the labor force" and "college graduate" independent? How do you know?

Exercise 18. Find some conditional probabilities. You know that a person is employed. What is the conditional probability that he or she is a college graduate? You know that a second person is a college graduate. What is the conditional probability that he or she is employed?